Networks III: Utility Analysis
Direct and Indirect Ecological Relations

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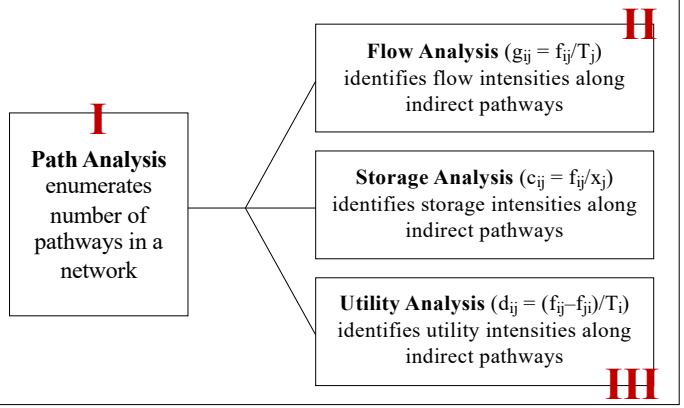
Network Environ Analysis



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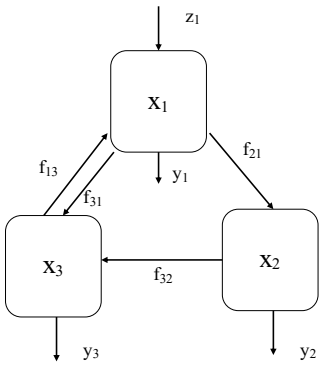

Connections

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Internal flows

$$F = \begin{bmatrix} 0 & 0 & f_{13} \\ f_{21} & 0 & 0 \\ f_{31} & f_{32} & 0 \end{bmatrix}$$

Input Output Storage

$$z = \begin{bmatrix} z_1 \\ 0 \\ 0 \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Dimensions of flows are $ML^{-2}T^{-1}$
Dimensions of storages are ML^{-2}

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Utility Analysis

- determines interaction types
- demonstrates network synergism and mutualism

Let $$d_{ij} = \frac{\left(f_{ij} - f_{ji}\right)}{T_i} = g'_{ij} - g_{ji}$$

- Normalized net flow between components

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Interactions

Transaction – transfer of energy or matter between two directly connected components

Relation – qualitative, value-oriented, direct or indirect interaction types. Nine possible interaction types

9 possible paired combinations using +, 0, and –

	+	0	–
+	(+,+)	(+,0)	(+,–)
0	(0,+)	(0,0)	(0,–)
–	(–,+)	(–,0)	(–,–)

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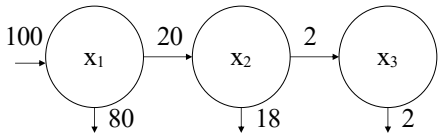
Three compartment food chain



Network utility analysis uses net flow between components

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad F = \begin{bmatrix} 0 & 0 & 0 \\ 20 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 100 \\ 20 \\ 2 \end{bmatrix}$$

$$d_{ij} = \frac{f_{ij} - f_{ji}}{T_i} \qquad D = \begin{bmatrix} {}^{0}/_{100} & {}^{-20}/_{100} & {}^{0}/_{100} \\ {}^{20}/_{20} & {}^{0}/_{20} & {}^{-2}/_{20} \\ {}^{0}/_{2} & {}^{2}/_{2} & {}^{0}/_{2} \end{bmatrix} \qquad D = \begin{bmatrix} 0 & -0.2 & 0 \\ 1 & 0 & -0.1 \\ 0 & 1 & 0 \end{bmatrix}$$

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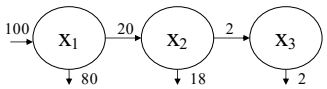
Direct Sign Matrix



$$D = \begin{bmatrix} 0 & -20/100 & 0 \\ 1 & 0 & -2/20 \\ 0 & 1 & 0 \end{bmatrix}$$

$$sgn(D) = \begin{bmatrix} 0 & - & 0 \\ + & 0 & - \\ 0 & + & 0 \end{bmatrix}$$

$$sgn(D) = \begin{bmatrix} 0 & - & 0 \\ + & 0 & - \\ 0 & + & 0 \end{bmatrix}$$

Direct relations – from comparing terms across the main diagonal:

$(sd_{21},sd_{12}) = (+, –) \rightarrow$ predation
$(sd_{32},sd_{23}) = (+, –) \rightarrow$ predation
$(sd_{31},sd_{13}) = (0, 0) \rightarrow$ neutralism

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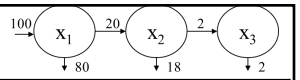
Integral Utility:



Utility: $U = I + D + D^2 + D^3 + D^4 + \ldots$

integral = initial input + direct + indirect

$$U = \sum_{m=0}^{\infty} D^m = (I - D)^{-1}$$

What is indirect relation between X1 and X3?

$$U = \begin{bmatrix} 0.846 & -0.154 & 0.015 \\ 0.769 & 0.769 & -0.077 \\ 0.769 & 0.769 & 0.923 \end{bmatrix}$$

$$sgn(U) = \begin{bmatrix} + & - & + \\ + & + & - \\ + & + & + \end{bmatrix}$$

$(sd_{21},sd_{12}) = (+, –) \rightarrow$ predation
$(sd_{32},sd_{23}) = (+, –) \rightarrow$ predation
$(sd_{31},sd_{13}) = (+, +) \rightarrow$ **mutualism**

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3-compartment food chain example

Relations Summary	Direct	Integral
Number + utilities	2	7
Number – utilities	2	2
+/ – sign ratio	1.00	3.5

$$\text{sgn}(D) = \begin{bmatrix} 0 & - & 0 \\ + & 0 & - \\ 0 & + & 0 \end{bmatrix} \qquad \text{sgn}(U) = \begin{bmatrix} + & - & + \\ + & + & - \\ + & + & + \end{bmatrix}$$

Direct Integral

Community-level relations are more positive than the direct relations that produced them: **This is network mutualism**.

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U gives nondimensional integral utility

To redimensionalize, pre-multiply diagonal throughflow vector

$$\mathrm{Y} = \hat{T}U = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0.846 & -0.154 & 0.015 \\ 0.769 & 0.769 & -0.077 \\ 0.769 & 0.769 & 0.923 \end{bmatrix}$$

$$= \begin{bmatrix} 84.6 & -15.4 & 1.54 \\ 15.4 & 15.4 & -1.54 \\ 1.54 & 1.54 & 1.85 \end{bmatrix}$$

$$\frac{\sum(+utility)}{\sum(-utility)} = \frac{149.59}{16.94} = 8.83$$

Community-level utility is more positive than the direct utility: **This is network synergism.**

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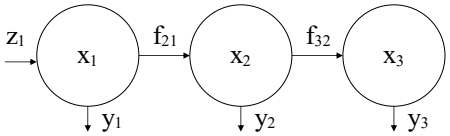
Three compartment food chain
General case



$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ f_{21} & 0 & 0 \\ 0 & f_{32} & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

Network utility analysis uses net flow between components

$$d_{ij} = \frac{f_{ij} - f_{ji}}{T_i}$$

$$D = \begin{bmatrix} 0 & -\frac{f_{21}}{T_1} & 0 \\ 1 & 0 & -\frac{f_{32}}{T_2} \\ 0 & 1 & 0 \end{bmatrix}$$

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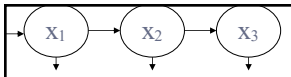

Integral (direct + indirect) relations

$$U = \frac{1}{1 + g_{21} + g_{32}} \begin{bmatrix} 1 + g_{32} & -g_{21} & g_{21}g_{32} \\ 1 & 1 & -g_{32} \\ 1 & 1 & 1 + g_{21} \end{bmatrix}$$

where $g_{ij}=f_{ij}/T_j$ and $0 < g_{ij} < 1$

$$\text{sgn}(U) = \begin{bmatrix} + & - & + \\ + & + & - \\ + & + & + \end{bmatrix}$$

$(sd_{21}, sd_{12}) = (+, -) \rightarrow$ predation
$(sd_{32}, sd_{23}) = (+, -) \rightarrow$ predation
$(sd_{31}, sd_{13}) = (+, +) \rightarrow$ mutualism

Network mutualism always occurs in this network structure

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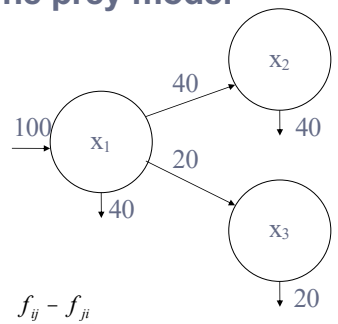
Two-predator, one prey model



$$d_{ij} = \frac{f_{ij} - f_{ji}}{T_i}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad F = \begin{bmatrix} 0 & 0 & 0 \\ 40 & 0 & 0 \\ 20 & 0 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 100 \\ 40 \\ 20 \end{bmatrix}$$

$$D = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix}$$

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Two-predator, one prey model

$$d_{11} = \frac{f_{11} - f_{11}}{T_1} \quad d_{12} = \frac{f_{12} - f_{21}}{T_1} \quad d_{13} = \frac{f_{13} - f_{13}}{T_1}$$

$$d_{21} = \frac{f_{21} - f_{12}}{T_2} \quad d_{22} = \frac{f_{22} - f_{22}}{T_2} \quad d_{21} = \frac{f_{23} - f_{32}}{T_2}$$

$$d_{31} = \frac{f_{31} - f_{13}}{T_3} \quad d_{32} = \frac{f_{32} - f_{23}}{T_3} \quad d_{11} = \frac{f_{33} - f_{33}}{T_3}$$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 40 & 0 & 0 \\ 20 & 0 & 0 \end{bmatrix} \quad T = \begin{bmatrix} 100 \\ 40 \\ 20 \end{bmatrix} \quad d_{ij} = \frac{f_{ij} - f_{ji}}{T_i}$$

$$D = \begin{bmatrix} 0 & -0.40 & -0.20 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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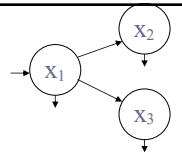
Direct Sign Matrix

$$\text{sgn}(D) = \begin{bmatrix} 0 & - & - \\ + & 0 & 0 \\ + & 0 & 0 \end{bmatrix}$$



$(sd_{21}, sd_{12}) = (+, -) \rightarrow$ predation
$(sd_{31}, sd_{13}) = (+, -) \rightarrow$ predation
$(sd_{32}, sd_{23}) = (0, 0) \rightarrow$ neutralism

Integral Sign Matrix

$$U = \begin{bmatrix} 0.625 & -0.250 & -0.125 \\ 0.625 & 0.75 & -0.125 \\ 0.625 & -0.25 & 0.875 \end{bmatrix} \quad \text{sgn}(U) = \begin{bmatrix} + & - & - \\ + & + & - \\ + & - & + \end{bmatrix}$$

$(sd_{21}, sd_{12}) = (+, -) \rightarrow$ predation
$(sd_{31}, sd_{13}) = (+, -) \rightarrow$ predation
$(sd_{32}, sd_{23}) = (-, -) \rightarrow$ competition

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Dimensional integral utility

$$Y = \hat{T}U = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 20 \end{bmatrix} \begin{bmatrix} 0.625 & -0.250 & -0.125 \\ 0.625 & 0.75 & -0.125 \\ 0.625 & -0.25 & 0.875 \end{bmatrix} = \begin{bmatrix} 62.5 & -25.0 & -12.5 \\ 25.0 & 30.0 & -5.0 \\ 12.5 & -5.0 & 17.5 \end{bmatrix}$$

	Direct matrix	Integral matrix
<u>Utility Summary</u>		
Sum + utilities	60	147.5
Sum – utilities	–60	–47.5
Benefit-cost ratio	1.00	**3.1**
<u>Relations Summary</u>		
Number + utilities	2	5
Number – utilities	2	4
+/ – sign ratio	1.00	1.25

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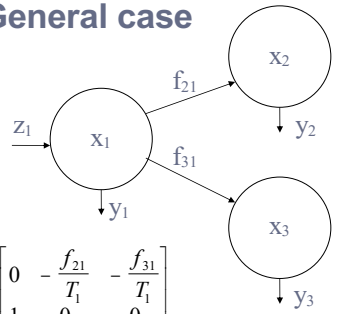
Two-predator, one prey model General case



$$D = \begin{bmatrix} 0 & -\frac{f_{21}}{T_1} & -\frac{f_{31}}{T_1} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ f_{21} & 0 & 0 \\ f_{31} & 0 & 0 \end{bmatrix}$$

$$T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

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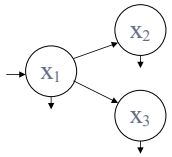
Direct Sign Matrix

$$\text{sgn}(D) = \begin{bmatrix} 0 & - & - \\ + & 0 & 0 \\ + & 0 & 0 \end{bmatrix}$$



$(sd_{21},sd_{12}) = (+, –) \rightarrow$ predation
$(sd_{31},sd_{13}) = (+, –) \rightarrow$ predation
$(sd_{32},sd_{23}) = (0, 0) \rightarrow$ neutralism

Integral Sign Matrix

$$U = \frac{1}{1 + g_{21} + g_{32}} \begin{bmatrix} 1 & -g_{21} & -g_{31} \\ 1 & 1+g_{31} & -g_{31} \\ 1 & -g_{21} & 1+g_{21} \end{bmatrix} \qquad \text{sgn}(U) = \begin{bmatrix} + & - & - \\ + & + & - \\ + & - & + \end{bmatrix}$$

$(sd_{21},sd_{12}) = (+, –) \rightarrow$ predation
$(sd_{31},sd_{13}) = (+, –) \rightarrow$ predation
$(sd_{32},sd_{23}) = (–, –) \rightarrow$ competition

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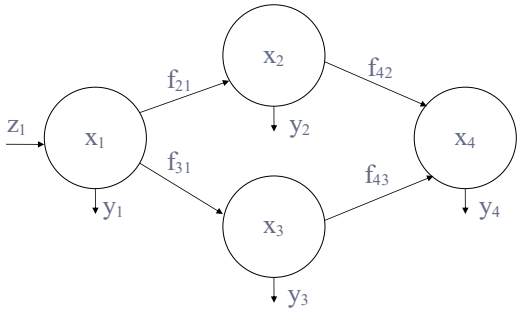
Two-predator, one prey plus top predator



$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 0 & 0 & 0 \\ f_{21} & 0 & 0 & 0 \\ f_{31} & 0 & 0 & 0 \\ 0 & f_{42} & f_{43} & 0 \end{bmatrix} \qquad T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

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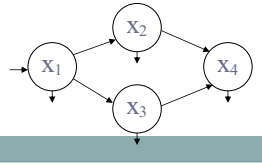
Direct Utility

$$D = \begin{bmatrix} 0 & -\frac{f_{21}}{T_1} & -\frac{f_{31}}{T_1} & 0 \\ 1 & 0 & 0 & -\frac{f_{42}}{T_2} \\ 1 & 0 & 0 & -\frac{f_{43}}{T_3} \\ 0 & \frac{f_{42}}{T_4} & \frac{f_{43}}{T_4} & 0 \end{bmatrix} \qquad \text{sgn}(D) = \begin{bmatrix} 0 & - & - & 0 \\ + & 0 & 0 & - \\ + & 0 & 0 & - \\ 0 & + & + & 0 \end{bmatrix}$$

Direct Relations

$(sd_{21},sd_{12}) = (+, -) \rightarrow$ predation
$(sd_{31},sd_{13}) = (+, -) \rightarrow$ predation
$(sd_{42},sd_{24}) = (+, -) \rightarrow$ predation
$(sd_{43},sd_{34}) = (+, -) \rightarrow$ predation
$(sd_{41},sd_{14}) = (0, 0) \rightarrow$ neutralism
$(sd_{32},sd_{23}) = (0, 0) \rightarrow$ neutralism



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Integral Relations

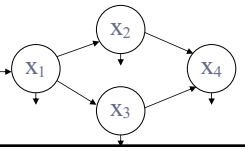
$$\text{sgn}(U) = \begin{bmatrix} + & - & - & + \\ + & + & - & ? \\ + & - & + & ? \\ + & ? & ? & + \end{bmatrix}$$

Relation type depends on function as well as structure

Three possible scenarios involving (su_{42}, su_{24}) and (su_{43}, su_{34})

$$\text{sgn}(U) = \begin{bmatrix} + & - & - & + \\ + & + & - & - \\ + & - & + & - \\ + & + & + & + \end{bmatrix} \quad \text{sgn}(U) = \begin{bmatrix} + & - & - & + \\ + & + & - & - \\ + & - & + & + \\ + & + & - & + \end{bmatrix} \quad \text{sgn}(U) = \begin{bmatrix} + & - & - & + \\ + & + & - & + \\ + & - & + & - \\ + & - & + & + \end{bmatrix}$$

Regardless of the scenario:
positive signs (10) > # negative signs (6)
Network Mutualism occurs



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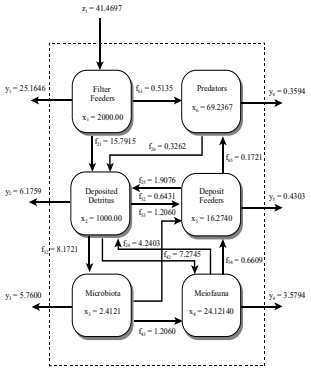
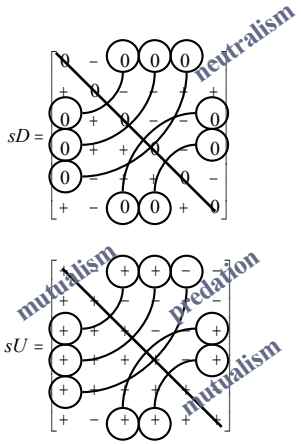
Oyster Reef Model





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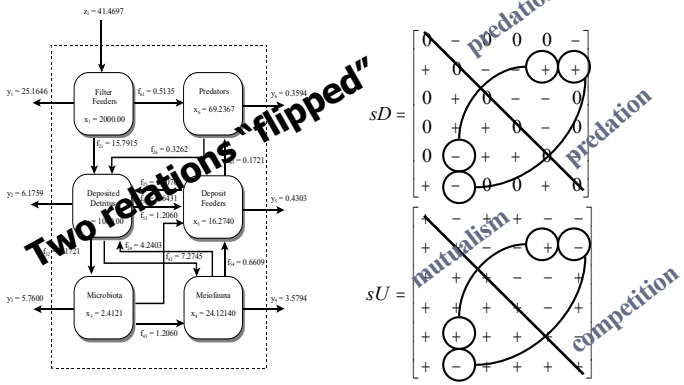
Oyster Reef Model



Two relations "flipped"

$$sD = \begin{bmatrix} 0 & - & 0 & 0 & - \\ + & 0 & - & + & + \\ 0 & + & 0 & - & 0 \\ 0 & + & + & 0 & 0 \\ 0 & - & + & + & 0 \\ + & - & 0 & + & 0 \end{bmatrix}$$

predation

predation

$$sU = \begin{bmatrix} + & - & + & + & - & - \\ + & + & - & + & + & - \\ + & + & + & - & - & + \\ + & + & + & + & - & + \\ + & + & + & + & + & - \\ + & - & + & + & + & + \end{bmatrix}$$

mutualism

competition

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Management applications of community relations

Bondavalli and Ulanowicz found that American Alligator feeding benefits 11 of its prey populations (e.g., frogs, mice, rats, and invertebrates) during wet season through indirect interactions.

Bondavalli C, Ulanowicz RE, 1999, Ecosystems, 49-63.

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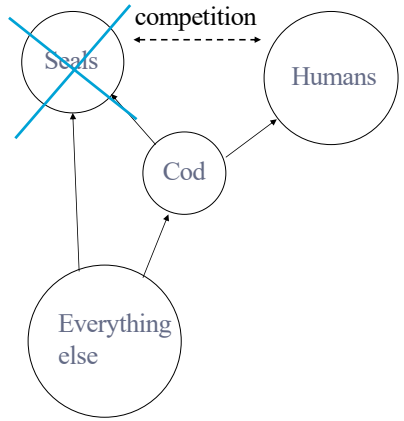
Management applications of community relations: Simplified fisheries model



From Yodzis, P, 1998, J. Animal Ecology, 635-658.

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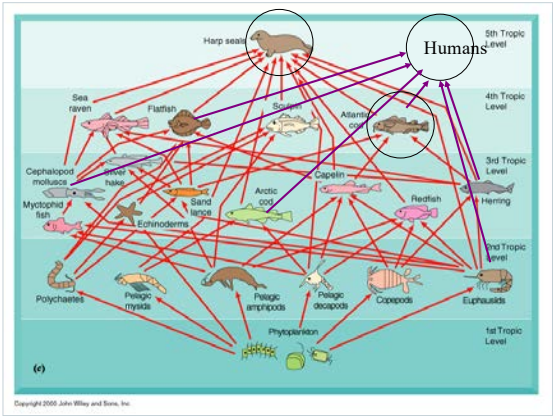
More realistic fisheries model



Yodzis found a greater probability for decrease in total fisheries yield after reduction in seal biomass. Single species management gave poorest results. Yodzis, P, 1998, J. Animal Ecology, 635-658.

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Conclusions of integral (system-level) relations

1) All ecosystem compartments are connected through indirect relations (no zeros in U matrix)
2) System-level relations can differ from direct observed relations (what you see, may not be what you get)
3) System-level relations usually exhibit mutualism over competition

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THANK YOU FOR YOUR ATTENTION

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